1 Assignment 3

We have seen that the oscillation probability of neutrinos are described by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - \sum_{i>j}^{n} \left[4\,\mathrm{Re}\left(A^{ij}_{\alpha\beta}\right) \sin^2 \frac{\Delta_{ij}}{2} - 2\,\mathrm{Im}\left(A^{ij}_{\alpha\beta}\right) \sin \Delta_{ij} \right], \tag{1.1}$$

with $A^{ij}_{\alpha\beta} \equiv U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}$, $\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{2E_\nu} = \frac{(m_i^2 - m_j^2)L}{2E_\nu}$ and n the number of neutrino states. In the standard three-neutrino framework, $n = 3$, and U is the PMNS mixing matrix (dropping Majorana phases):

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{pmatrix}$$

Now, we will explore how this probability is modified with a non-Hermitian Hamiltonian, which can be used to describe neutrino decay or absorption.

(**3 points**) 1. Introduce a non-Hermitian Hamiltonian,

$$\hat{H} = \hat{H}_0 - \frac{i\hat{\Gamma}}{2}, \tag{1.2}$$

with $\hat{H}_0$ being the vacuum term and $\hat{\Gamma}$ describing a leakage of the states out of the system. When $\hat{\Gamma}$ is diagonal in the mass basis, that is, $\hat{\Gamma}\,|\nu_i\rangle = \Gamma_i\,|\nu_i\rangle$, then we can interpret $\Gamma_i \geq 0$ as the decay width of the state $|\nu_i\rangle$. The full time-evolution of a mass eigenstate contains a complex as well as a real exponential to account for the loss of the state over time, $\langle \nu_i | \nu_i(t) \rangle = e^{-iE_i t} e^{-\Gamma_i t/2}$. Therefore, if the state $|\nu_i\rangle$ is produced at $t = 0$, the probability of finding it at a later time t is given by $P(\nu_i \to \nu_i) = e^{-\Gamma_i t}$, which indicates it is undergoing some decay process to a state outside of our system. Show that

$$P(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 e^{-\Gamma_i t} + \sum_{i>j} 2e^{-\gamma_{ij}} \left[\mathrm{Re}(A^{ij}_{\alpha\beta}) \cos \Delta_{ij} + \mathrm{Im}(A^{ij}_{\alpha\beta}) \sin \Delta_{ij} \right], \tag{1.3}$$

where $\gamma_{ij} = (\Gamma_i + \Gamma_j)t/2$.

(**1 point**) 2. Explain the two limits: $\Gamma_i \to 0, \forall i = 1, 2, 3$ and $\Gamma_i t \gg 1, \forall i = 1, 2, 3$.

(**3 points**) 3. Write down the ν_e survival probability $P(\nu_e \to \nu_e)$ in the three-neutrino framework in the limit of $\Delta_{31} \gg 1$, so that Δ_{31} oscillations have averaged out. Write everything in terms of of the PMNS mixing angles θ_{12}, θ_{13}, and θ_{23}. Given that we have seen that θ_{13} is small, drop terms of $\mathcal{O}(\sin^4 \theta_{13})$. Show that *oscillations* can be observed if $\Gamma_3 t \gg 1$, but not if $\Gamma_1 t \gg 1$ or $\Gamma_2 t \gg 1$.

(**1 points**) 4. The KamLAND experiment observed a striking ν_e disappearance oscillation signal using neutrinos from reactors located at an average distance of $L = 180$ km away and

typical energy of $E_\nu = 4$ MeV. Given that, set a lower limit on the lifetime τ of ν_1 and ν_2 given that $\Gamma_i = \frac{m_i}{E_\nu}\frac{1}{\tau_i}$. Assume $m_1 \sim 0.01$ eV and $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV^2 and express your answer in seconds. (Hint: your limit can arise from the requiremenet that, e.g., $\Gamma_i L \ll 1$.)

(**2 point**) 5. Now interpret Γ_i as the rate of absorption of ν_i due to interactions with the medium as it propagates through the Earth. With the same requirement as above, set a limit on the size of the neutrino absorption cross section on nucleon, σ_i, given that $\Gamma_i = \sigma_i n_{\text{nucleons}}$, with n_{nucleons} the number density of nucleons in the Earth. Express your answer in cm^2.

(**Bonus 2 points**) 6. By extending Eq. (1.3) to antineutrinos, write down the CP asymmetry $A_{e\mu} = P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$ in the limit of $\Delta_{21} \to 0$ and neglecting any term of $\mathcal{O}(\theta_{13}^2)$ or higher. Write everything in terms of of the PMNS mixing angles θ_{12}, θ_{13}, and θ_{23}, as well as the sine of the CP-violating phase $\sin\delta_{\text{CP}}$. In what limits of $\Gamma_i t$ can CP violation be observed?