1 Low-scale seesaw models (5 points)

This exercise will explore low-scale variants of the seesaw mechanism. The idea will be to turn the seesaw on its head and explore symmetries to make neutrino masses small. Low-scale seesaws (sometimes referred to as double, extended, linear, or inverse seesaws) introduce a pair of neutral lepton per neutrino family (call them ν_R and ν'_R, for example). In a single-generation scenario, the neutrino mass matrix becomes

$$H_{\text{int}} = \begin{pmatrix} |\nu_L\rangle & |\bar{\nu}'_L\rangle & |\bar{\nu}_L\rangle \end{pmatrix} \begin{pmatrix} 0 & m_D & \varepsilon \\ m_D & \mu' & M \\ \varepsilon & M & \mu \end{pmatrix} \begin{pmatrix} \langle\bar{\nu}_R| \\ \langle\nu'_R| \\ \langle\nu_R| \end{pmatrix} + \text{h.c.} \tag{1.1}$$

1. The quantum numbers of the neutral leptons are $L(\nu_L) = +1$, $L(\nu_R) = +1$, and $L(\nu'_R) = -1$. Out of all the parameters in the matrix above, which ones violate lepton number?

2. Calculate the determinant of this matrix. What happens when all the lepton-number-violating (LNV) parameters go to zero? What does this imply for light neutrino masses?

3. Keeping all these lepton-number-violating parameters zero and assuming the Dirac mass m_D is much smaller than all other parameters, diagonalize the matrix above and analyze what happens with the spectrum of massive particles. What kind of particles do you find?

4. Turn on only a single LNV parameter, say $\mu \neq 0$, at a time and diagonalize the matrix assuming $m_D \ll$ LNV parameters $\ll$ anything else. (you may use Mathematica, Wolfram Alpha, or Python, but you don't have to). You can assume all parameters are real and positive.

   How do the light neutrino masses depend on the LNV parameters in this case? To explain the smallness of neutrino masses, do you expect lepton number to be a good or a badly broken symmetry in these scenarios?

5. What kind of particle are the heavy neutral leptons in our spectrum? Dirac, quasi-Dirac, or Majorana fermions?

2 Heavy neutrino decay and detection (5 points)

Let us now look at how you could search for a single heavy neutral lepton (HNL) in an accelerator experiment. Let's call this HNL ν_4 with mass m_4 and assume it mixes predominantly with the muon neutrino flavor via a mixing angle $|U_{\mu 4}|^2$. In other words, every time a muon neutrino is produced, there is a probability $|U_{\mu 4}|^2$ that it is actually a heavy neutrino ν_4 instead. Diagramatically, this can be represented as:

$$\nu_\mu \;\longrightarrow \underset{U_{\mu 4}}{\times} \longrightarrow\; \nu_4$$

This exercise will guide you through an estimate of the signal event rate for HNLs produced by pion decay-at-rest, which subsequently decay inside a large-volume detector far away. Consider a spallation source with $\mathcal{O}(\text{GeV})$ protons hitting a dense target. The number of pions produced at this source is related to the number of protons on target by $N_\pi \sim 10\% \times N_{\text{POT}}$. As soon as they are produced, they lose energy and stop inside the target, decaying at rest via $\pi^+ \to \mu^+ \nu_\mu$. Typical accelerators produce $N_{\text{POT}} \sim 10^{23}$ protons on target per year.

1. Using the mixing line drawn above, draw a Feynman diagram for the production of HNLs from pion decay $\pi^+ \to \mu^+ \nu_4$. Now also draw a decay diagram for the HNL to decay via $\nu_4 \to \nu_\mu e^+ e^-$.

2. The branching ratio for $\pi^+ \to \mu^+ \nu_4$ is approximately

$$\mathcal{B}(\pi^+ \to \mu^+ \nu_4) \sim |U_{\mu 4}|^2 \mathcal{B}(\pi^+ \to \mu^+ \nu_\mu), \tag{2.1}$$

   where $\mathcal{B}(\pi^+ \to \mu^+ \nu_\mu) \simeq 100\%$. Here, $|U_{\mu 4}|^2$ is the mixing angle of the heavy neutrino with the muon neutrinos [1].

   Using that branching and the number of pions produced per POT and the fact that the decay to HNLs is isotropic, what is the flux of HNLs at a distance L from the target per year of operation? Account for the lifetime of the HNL by multiplying this flux by the probability for ν_4 to survive the distance L without decaying.

   Now estimate the number of HNLs that do decay within a region of volume $V = \lambda^3$ located at a distance L from the source. Assume $\lambda \ll L$.

3. Knowing that the muon lifetime is determined by the decay rate $\Gamma = G_F^2 m_\mu^5 / 192\pi^3$, estimate the analogous decay rate for $\nu_4 \to \nu_\mu e^+ e^+$.

4. Putting all of this together, write down a formula for the number of HNL decays within this detector of size V. You can assume the HNL decays only via $\nu_4 \to \nu e^+ e^-$

[1] Note this is analogous to θ^2 in exercise 1, except that it is now phrased in a 4-neutrino mixing picture and makes use of the relation $|\nu_\alpha\rangle = \sum_i^4 U^*_{\alpha i} |\nu_i\rangle$

with a rate Γ_{ν_4}. Expand this formula assuming $\Gamma_{\nu_4} L$ and $\Gamma_{\nu_4}\lambda$ are very small. How does the signal event rate depend on the decay rate Γ_{ν_4}?