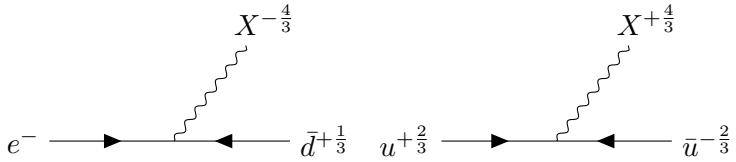
Assignment 2. Where is the new physics?

Inspecting all the SM couplings and possible vertices, we find that there is no interaction vertex that changes a quark into a lepton or vice-versa. This is a consequence of another accidental symmetry of the SM: baryon number, $B(p^+) = B(n) = 1$ and $B(q) = 1/3$ with $B(e^-) = B(\nu) = 0$. In Grand-Unified Theories (GUTs), all the SM forces are expected to unify at a very high energy scale. At these scales, one expects new mediators that treat quarks and leptons equally to exist. In many cases, they end up interacting with both quarks and leptons at the same vertex. For example, consider a new heavy charged mediator $X^{\pm\frac{4}{3}}$ ($Q_{\rm EM}(X) = \pm 4/3$) that can couple to both a quark and a charged lepton on the same vertex (often referred to as a leptoquark):



(vertex: quarks and lepton interaction with the leptoquark $X^{\pm 4/3}$. Note the clashing arrows, indicating the violation of some quantum number.)

Clearly, these hypothetical leptoquark interactions violate the accidental baryon number conservation law.

1. Draw a possible Feynman diagram for proton decay, $p^+ \to e^+ + \pi^0$, at the quark level. (Hint: In addition to potential spectator quarks line(s), your diagram should contain exactly one X particle. Recall that like the Z, $W^\pm$, or γ, at each one of its interaction vertices, you should have only one boson and two fermions.)

2. The proton decay process violates both baryon number and lepton number individually. What combination of B and L remains conserved in this process?

3. Through naive dimensional analysis, give an order of magnitude expression for the proton lifetime τ_p in terms of the mass of the new mediator m_X and the coupling constant g_X of this new interaction. (Hint: take the proton mass to be $m_p \simeq 938$ MeV and neglect all other energy scales like the electron and pion masses.)

4. Large volume neutrino detectors like the Super-Kamiokande and the future Hyper-Kamiokande can be used to search for proton decay. The signature is a positron and two photons (from the $\pi^0 \to \gamma\gamma$ decay) originating from a single point in the detector. It is so clean that there is essentially no background to this process. If Hyper-Kamiokande has a total volume of about 0.4 Mton of water, how many free protons does it contain (free = not bound to a *nucleus*)? If they do not observe even a single spontaneous decay of a proton into $e^+\pi^0$ in a total of 10 years of operation, what could we deduce about the minimum proton lifetime? Express your answer in years and also in units of the age of the universe, $t_U \simeq 13.8$ Gyr. (PS: you should know about the fun fact that 1 year $\simeq \pi \times 10^7$ s.)

5. Using your answer above to question (2), what does this limit on the proton lifetime imply for the scale of new physics m_X? You can assume that all couplings are $g_X \sim \mathcal{O}(1)$.

6. To improve our reach in m_X and get to the Planck scale ($M_P \simeq 10^{19}$ GeV), how much longer would you need to run Hyper-Kamiokande? (PS: the Planck scale is often considered to be the scale where *quantum gravity* effects become important.)
