Assignment 1. The cosmic neutrino background (CνB) was formed when neutrinos decoupled from the primordial plasma at a temperature of about $T_{\text{early}} \simeq 1$ MeV. This means that at that temperature, neutrinos were finally able to propagate freely and, on average, were no longer scattering on other background particles in reactions such as $\nu_e + n \leftrightarrow e^- + p^+$. Since then, these neutrinos continued to freely propagate everywhere in the universe until reaching us today, redshifting down in energy. Today, they form a collisionless gas of temperature $T_{\text{now}} = T_{\text{early}}/(1 + z_{\text{C}\nu\text{B}}) \simeq 1.95$ K, where $z_{\text{C}\nu\text{B}} \simeq 10^9$ is called the *redshift*. (Contrast this with the cosmic microwave background (CMB), which decoupled at a much later time, around $T_{\text{CMB-early}} \simeq 0.25$ eV and $z_{\text{CMB}} \simeq 1100$.)

Note that all weakly-interacting neutrinos, namely $|\nu_L^\alpha\rangle$ and $|\bar{\nu}_R^\alpha\rangle$ with $\alpha \in \{e, \mu, \tau\}$, were produced with the same proportions in the early universe and that the total number of neutrinos (of all types) in our ambient today is about $n \sim 330\ (\nu + \bar{\nu})/\text{cm}^3$.

1. Assuming that all neutrinos have a mass of about $m_\nu \simeq 0.1$ eV and using the CνB temperature above, estimate i) the typical kinetic energy $E_{\text{kin}} = E - m \simeq T_{\text{now}}$ of neutrinos in the CνB today in eV, and ii) the momentum p of a single neutrino state today in eV. What is its typical velocity as a fraction of the speed of light?

2. What is the flux of these neutrinos in neutrinos/cm^2/s? Compare this with the flux of neutrinos emitted by the Sun, $\Phi(\text{solar}) = 7 \times 10^{10}\ \nu_e/\text{cm}^2/\text{s}$, which have much higher energies, $E_\nu \simeq 1$ MeV.

3. **Picking a single neutrino flavor α, write the initial chiral neutrino eigenstates in terms of helicity states. What does this simplify to for today's CνB energies? Assuming neutrinos are Dirac particles, in what proportion are the different degrees of freedom (different types of propagating particles) to be found in the CνB today if neutrinos are Dirac? How about if they are Majorana?** (Hint: note that for a given neutrino flavor, Dirac neutrinos have four and Majorana neutrinos have two degrees of freedom. In terms of the interaction states, these are $|\nu_L^\alpha\rangle$, $|\nu_R^\alpha\rangle$, $|\bar{\nu}_L^\alpha\rangle$, and $|\bar{\nu}_R^\alpha\rangle$ for Dirac neutrinos, and $|\nu_L^\alpha\rangle$ and $|\bar{\nu}_R^\alpha\rangle$ for Majorana. In terms of the helicity states (the true propagating degrees of freedom): $|\nu_-^\alpha\rangle$, $|\nu_+^\alpha\rangle$, $|\bar{\nu}_-^\alpha\rangle$, and $|\bar{\nu}_+^\alpha\rangle$ for Dirac, and just $|\nu_-^\alpha\rangle$ and $|\nu_+^\alpha\rangle$ for Majorana (no bar-ed states, recall Eq. (3.11)!).)

4. PTOLEMY is a project that will attempt to detect the cosmic neutrino background via the *neutrino capture* reaction on tritium, namely $\nu_e + {}^3\text{H} \to {}^3\text{He} + e^-$. Draw a Feynman diagram for this process (include the spectator particles in your diagram if drawing the nucleon or quark-level diagrams). What neutrino interaction state is allowed to be captured (meaning, $|\nu_e^L\rangle$ vs $|\nu_e^R\rangle$ vs $|\bar{\nu}_e^L\rangle$ vs $|\bar{\nu}_e^R\rangle$, etc)? (Hint: think about charge conservation to exclude some possibilities, and think about energy conservation in this process to exclude others.)

5. If PTOLEMY employed 100 grams of tritium, how many neutrinos would you expect to interact in that 100 g sample over the course of a year? Show your workings for

this question.
(Hint: Take the neutrino capture cross section to be

$$\sigma_{\nu_e-\text{capture}} \sim \frac{G_F^2 m_\nu m_p}{2\pi}, \tag{4.8}$$

with $G_F = 1.166 \times 10^{-5}$ GeV^{-2} and $m_p \simeq 938$ MeV the proton mass. You will also need Avogadro's number, $N_A = 6.022 \times 10^{23}$ mole^{-1}, and the molar mass of tritium, $M(^3\text{H}) \simeq 3$ g/mole.)

6. How many tritium beta decay events occur in the same time period? (Hint: the half-life of tritium is $t_{1/2} \simeq 12.3$ years.)

7. What is the largest energy the electron may carry in the beta decay process, i.e., what is the Q-value of this reaction? How about in the neutrino capture process? Is it possible to see this signal on top of the beta-decay backgrounds? (Hint: the nuclear masses of tritium and He3 are: $m(^3\text{He}) \simeq 2808.391$ MeV and $m(^3\text{H}) \simeq 2808.921$ MeV. Think about the energy conservation in the process – the electron takes the most amount of energy when all other final state particles are at rest. You can assume that the CνB is also at rest.).

8. **Does the signal rate (ν_e) capture change if neutrinos are Dirac or Majorana particles?**
