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Particle Physics Beyond the Standard Model

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Resources Every particle physicist needs a reference to remember all the masses, lifetimes, and properties of the zoo of particles we know. Fortunately, there are some great resources online with all the information you need (for free!). The most standard reference is the Particle Data Group (PDG), which compiles all the relevant particle physics results to date in an easy-to-access way. You can also order your own booklet if you'd like a physical copy.

- https://pdglive.lbl.gov/
- The PDG also has several review pages on particle physics and cosmology: https://pdg.lbl.gov/2023/reviews/contents_sports.html — these are more advanced though.

Other useful websites you should know about for literature review:

1. INSPIRE-HEP: https://inspirehep.net/
2. arXiv: https://arxiv.org

1 Reminders and Natural Units

Particle physicists like to express physical quantities in units that are *not* defined by human standards, like meters, seconds, or grams. Instead, we define new "natural units" that: 1) are based on universal and fundamental laws of nature and 2) that make our equations as simple as possible. This approach treats dimensionful constants (are they even constant if they depend on your choice of units?) as conversion factors between different physical quantities.

As you learned in your Special Relativity course, the speed of light in vacuum,

$$c = 299\,792\,458 \text{ m/s} \simeq 1 \text{ foot/ns}, \tag{1.1}$$

is a constant for all observers; therefore, it is as universal a quantity as we could hope for. In natural units, we enforce c to be unity, such that

$$c \stackrel{!}{=} 1 \implies 1 \text{ s} = 299\,792\,458 \text{ m}. \tag{1.2}$$

That is, by setting $c = 1$, we have simply stated that meters and seconds are equivalent measures of something (call it a space-time distance, for instance), and that it takes a lot of meters to have a comparable "space-time" second. By imposing Eq. (1.2), we have effectively reduced the number of fundamental units we need to keep track of by one: you may now use only meters or only seconds to fully specify a physical quantity.

As a consequence, speed or velocity are dimensionless quantities since their magnitudes are always interpreted as a fraction of the speed of light, which is just unity ($c = 1$).

The relativistic energy for a particle with mass m and momentum p as

$$E^2 = m^2c^4 + p^2c^2. \tag{1.3}$$

In natural units,

$$E^2 = m^2 + p^2. \tag{1.4}$$

The equivalence between mass, momentum, and energy becomes much clearer in this way: E, m, and p can all be expressed in terms of energy units.

Another universal constant we can leverage to simplify our units is Planck's constant h (in particular, its reduced form will be make less of a mess):

$$\hbar = \frac{h}{2\pi} = 1.054571817 \times 10^{-34}\ \text{J.s} = 6.582119569 \times 10^{-16}\ \text{eV/Hz}, \tag{1.5}$$

where we used the definition of an electronvolt in the second equality (1 eV $= 1.602176634 \times 10^{-19}$ J is the energy acquired by an electron in an electric potential of one volt. Hence, electronvolt is a unit of energy.) By setting

$$\hbar \overset{!}{=} 1 \implies 1\ \text{Hz} = \text{s}^{-1} = 6.582119569 \times 10^{-16}\ \text{eV}. \tag{1.6}$$

we make it clear that $\hbar$ relates energy to frequency (or inverse time) in a very fundamental way. This follows the physics since the energy of a photon is given by

$$E = h\nu = \hbar\omega = \frac{2\pi\hbar}{T} = \frac{2\pi\hbar c}{\lambda}, \tag{1.7}$$

where ν is the frequency of the photon, $\omega = 2\pi\nu$ is its angular frequency, T is its period, and λ is its wavelength. In fact, from all equalities above, you can deduce that $\hbar = 1$ and $c = 1$ put energy, frequency, time, and distance all on the same footing.

We started with three units: energy (eV), time (s), and distance (m). By setting $\hbar = 1$ and $c = 1$, we imposed two independent constraints on the system, leaving just one independent fundamental unit. We will pick it to be energy.

By convention, when working with natural units, we typically express all quantities in electronvolt (eV) or powers thereof. When necessary, we convert back to meters, seconds, or whatever else, by making use of the definition of $\hbar$ in Eq. (1.5), c in Eq. (1.2), or other convenient factors such as:

$$\hbar c \simeq 197\ \text{MeV.fm} = 0.197\ \text{GeV.fm} = 1.97 \times 10^{-5}\ \text{eV.cm}. \tag{1.8}$$

The reduced Compton wavelength of a massive particle is

$$\lambda\!\!\!^{-} = \frac{\hbar}{m_e c}. \tag{1.9}$$

What is $\lambda\!\!\!^{-}$ for electrons, $m_e = 0.511$ MeV? How about for a proton, $m_p = 938$ MeV? Express your answers in femtometers (1 fm $= 10^{-15}$ m).

The mass of a proton is approximately $m_p \simeq 938$ MeV. Using natural units, find the mass of a proton at rest in grams. How many protons in a gram?

The Hubble constant is $H_0 \simeq 75$ (km/s)/Mpc. What distance does H_0^{-1} correspond to in Mpc? This is approximately the size of our causal horizon. What is H_0 in eV?

The above is the bulk of what we will need for our calculations, but there are two more examples that are worth mentioning. In Maxwell's equations, we also have the permittivity of free space ϵ_0:

$$\epsilon_0 = 8.854\,187\,8128 \times 10^{-10}\ \mathrm{C}^2/(\mathrm{N}\cdot\mathrm{m}^2) \simeq 1.418 \times 10^{-32}\ \mathrm{C}^2/(\mathrm{eV}\cdot\mathrm{cm}). \tag{1.10}$$

This too, we will set to unity, $\epsilon_0 = 1$. In this case, the new constraint relates, for example, Coulombs (C), electronvolts (eV), and centimeters (cm). From $c = \hbar = 1$ and Eq. (1.8), we already know that centimeters and electronvolts are inversely related such that their product comes out dimensionless. Therefore, Setting $\epsilon_0 = 1$ has trivialized Coulombs as a unit and has set it to a dimensionless value of $\mathrm{C} \simeq 5.28 \times 10^{-19}$.

In natural units, electric charge, usually expressed in terms of the elementary charge $e = 1.602176634 \times 10^{-19}$ C, is dimensionless. From Eq. (1.10), you can show that

$$e \simeq 0.303 \implies \alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}. \tag{1.11}$$

Now, that's a constant! (Or is it?)

Let us connect this permittivity discussion to a familiar problem. Consider the energy ΔE gained by an electron of elementary charge $q_1 = e$ in an electric potential V sourced by a point charge $q_2 = Qe$ at distance r away. In natural units, it is simply given by

$$\Delta E = q_1 \cdot V = \frac{q_1 q_2}{4\pi r} = \frac{\alpha Q}{r}. \tag{1.12}$$

This should feel more natural, the energy gained depends purely on the distance proportionally to dimensionless quantities: the charges and the fine-structure constant $\alpha \simeq \frac{1}{137}$. The latter appears here to characterize the strength of the electromagnetic interaction.

A final example is Boltzmann's constant k_B:

$$k_B = 1.380649 \times 10^{-23}\ \text{J/K} = 8.617333262 \times 10^{-5}\ \text{eV/K}. \tag{1.13}$$

Recall that the typical kinetic energy of a particle in an ideal gas of temperature T is $E \sim 3k_BT/2$. By setting $k_B = 1$, we find that the temperature is simply a measure of energy.

What is the typical energy of a photon in the Comic Microwave Background given that its temperature is $T \simeq 2.7$ K? Express your answer in eV. How does this compare to the typical energy of a photon in visible light, which has a wavelength of about 500 nm? And to the ground state energy of the hydrogen atom?

The typical energy of a photon in the Cosmic Microwave Background (CMB) can be estimated using the relation $E \sim k_BT$. Given that $T \simeq 2.7$ K, we find

$$E \sim k_BT \simeq 8.617333262 \times 10^{-5}\ \text{eV/K} \times 2.7\ \text{K} \simeq 2.33 \times 10^{-4}\ \text{eV}. \tag{1.14}$$

In comparison, the energy of a photon in visible light with a wavelength of about 500 nm is given by

$$E \sim \frac{hc}{\lambda} \simeq \frac{(4.135667696 \times 10^{-15}\ \text{eV s})(3 \times 10^{10}\ \text{cm/s})}{500 \times 10^{-7}\ \text{cm}} \simeq 2.48\ \text{eV}. \tag{1.15}$$

Finally, the ground state energy of the hydrogen atom is approximately $E_1 \simeq -13.6$ eV.

Energy regimes. In particle physics, we often classify particles as non-relativistic or ultra-relativistic depending on the relative size of their mass m and momentum p.

In the non-relativistic regime, $m \gg p$ and

$$E = \sqrt{m^2 + p^2} = m\sqrt{1 + \frac{p^2}{m^2}} \simeq m + \frac{p^2}{2m} + \cdots . \tag{1.16}$$

As expected, the leading contribution to this particle's energy is its mass. The term $p^2/2m$ is then just the classical expression for its kinetic energy.

In the ultra-relativistic regime, $m \ll p$ and

$$E = p\sqrt{1 + \frac{m^2}{p^2}} \simeq p + \frac{m^2}{2p} + \cdots . \tag{1.17}$$

The energy is dominated by the momentum piece with a correction of size $m^2/2p$. The latter may be less familiar, but it will show up quite often in the world of particle physics, especially in neutrino.

Moving Waves. The plane wave wavefunction is given by

$$\psi(t,\vec{x}) \propto e^{-i(\omega t-\vec{k}\cdot\vec{x})}. \tag{1.18}$$

This wave has a well defined momentum $\vec{p}=\hbar\vec{k}$, but it is not localized in space. It is not normalizable and cannot be interpreted as a physical particle. Despite that, it can still be useful when describing certain phenomena that do not depend on position or when dealing with asymptotic states, which is often the case in high-energy collisions. We usually care about the incoming and outgoing states in the (practically) infinite past and future. Particle physicists rarely have to resort to a more sophisticated description, but it will come up in our discussion of neutrino oscillations, so we review it below.

One such description is a wave packet built from a superposition of plane waves of different momenta. A one-dimensional gaussian wavepacket, for instance, could start out looking like

$$\psi(t=0,x) = g(x,x_0,\sigma_x)e^{ik_0x}, \tag{1.19}$$

with k_0 some characteristic momentum (but not the only momentum of this wave) and

$$g(x,\mu,\sigma) = \frac{1}{(2\pi)^{\frac{1}{4}}\sqrt{\sigma}}e^{-\frac{(x-\mu)^2}{4\sigma^2}}. \tag{1.20}$$

You can convince yourself that this wavefunction is normalizable, $\int \mathrm{d}x|\psi(x)|^2=1$. Fourier transforming this wavefunction, we get all momentum modes

$$\widetilde{\psi}(t=0,k) = g(k,k_0,\sigma_x^{-1})e^{ix_0(k-k_0)}. \tag{1.21}$$

As expected, this is also a gaussian, now with a width such that $\sigma_k=\frac{1}{2}\sigma_x^{-1}$. Following the uncertainty principle, the more localized this wave packet is in position space (smaller σ_x), the more spread out it is in momentum space (larger σ_k).

The time dependence is then determined by

$$\psi(t,x) = \int \frac{\mathrm{d}k}{2\pi}\,\widetilde{\psi}(t,k)\,e^{-i(\omega(k)t-kx)}. \tag{1.22}$$

Since the wavefunction $\widetilde{\psi}(t,k)$ is peaked around k_0, we can Taylor expand it as

$$\omega(k) \simeq \omega_0 + (k-k_0)v_g + \cdots, \tag{1.23}$$

where $\omega_0\equiv\omega(k_0)$ and we defined the group velocity

$$v_g \equiv \left.\frac{\partial\omega}{\partial k}\right|_{k_0}. \tag{1.24}$$

Plugging this back in the integral, we find

$$\psi(t,x) \simeq e^{-i(\omega_0 t-k_0x)}\int \frac{\mathrm{d}k}{2\pi}\,\widetilde{\psi}(t,k_0)\,e^{-i(k-k_0)(v_gt-x)}. \tag{1.25}$$

The prefactor is just a plane wave with momentum k_0 and energy ω_0. More importantly, this plane wave is now modulated by the integral over momentum modes. The latter is just a function of x and describes the spread of the wave packet in position space.

Actually perform the integral in Eq. (1.25) keeping terms of order $(k-k_0)^2$ in the expansion of $\omega(k)$,

$$\omega(k) \simeq \omega_0 + v_g(k-k_0) + \tfrac{1}{2}\,\omega_0''(k-k_0)^2.$$

(Gaussian integrals will be very useful in QFT classes.) Stare at it long enough to convince yourself that the wave packet moves with group velocity v_g and spreads over time if $\omega_0'' \neq 0$. That is, if the dispersion relation is not constant or linear in k, as is the case for our relativistic particles, then the wave packet will change over time.

Group velocity. Specifying the energy-momentum relation $E(p)$ for a particle is equivalent to specifying its dispersion relation $\omega(k)$. As evident from the above, the phase velocity and the group velocity of a wave are not necessarily the same. In particle physics we are usually more interested in the group velocity, which is the velocity associated with the wavepacket, where the particle is most likely to be found. While the phase velocity is simply $v_p = \frac{\omega}{k}$, the group velocity is

$$v_g \equiv \frac{\partial\omega}{\partial k} = \frac{\partial E}{\partial p} = \frac{p}{E}. \tag{1.26}$$

While v_p can exceed the speed of light, the group velocity cannot, $v_g \leq 1$.

Some special cases of the group velocity are:

- Massless: $E = p \;\Rightarrow\; v_g = 1$.
- Particle at rest: $p = 0 \;\Rightarrow\; v_g = 0$.
- Non-relativistic: $v_g = \frac{p}{E} \simeq \frac{p}{m}\left(1 - \frac{p^2}{2m^2}\right)$.
- Ultra-relativistic: $v_g = \frac{p}{E} \simeq 1 - \frac{m^2}{2p^2}$.

Racing particles. Consider two particles with common momentum p but different masses m_1 and m_2, moving with group velocities v_1 and v_2. The difference in group velocities is given by $\Delta v \equiv v_1 - v_2$ and the lag between their positions is given by $\Delta x \equiv \Delta v\, t$. In the non-relativistic regime,

$$\Delta v \equiv v_1 - v_2 \simeq \frac{p}{m_1} - \frac{p}{m_2} = p\,\frac{m_2 - m_1}{m_1 m_2}. \tag{1.27}$$

The ultra-relativistic regime will be very useful later. It is given by

$$\Delta v \equiv v_1 - v_2 \simeq -\frac{m_1^2 - m_2^2}{2p^2} = \frac{m_2^2 - m_1^2}{2p^2}. \tag{1.28}$$

Defining $\Delta m_{21}^2 = m_2^2 - m_1^2$, the lag experience by particle 2 ($m_2 > m_1$) is given by

$$\Delta x \equiv \Delta v\, t \simeq \frac{\Delta m_{21}^2}{2p^2}\, t. \tag{1.29}$$

Only differences in m^2 matter at leading order for highly relativistic particles with the same p.

Assume instead that the two racing particles have the same energy, not momentum. What can we say about the difference between their group velocities and the resulting lag? How does this compare with the expression above?