3 Spin, helicity, and chirality

Having discussed chirality above, we now have to turn to the closely related concepts of spin and helicity. The spin of a fermion is an intrinsic form of angular momentum that is quantized in units of $\hbar/2 = 1/2$. The total spin operator is defined as

$$\mathbf{S} = \frac{\boldsymbol{\sigma}}{2},$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The total spin operator and its components form a Lie algebra with the commutation relations

$$[\hat{S}_i, \hat{S}_j] = i\epsilon_{ijk}\hat{S}_k.$$

Recall also that for any spin-s particle,

$$\hat{\mathbf{S}}^2 \left|s, m\right\rangle = s(s+1) \left|s, m\right\rangle, \qquad \hat{S}_z \left|s, m\right\rangle = m \left|s, m\right\rangle, \quad m \in \{-s, -s+1, \ldots, s\}.$$

This set of relations are in fact quite general and come from the fact that the spin operators form the Lie algebra of SU(2). This will also be the case for weak isospin, which has nothing to do with spin, but is described by the same mathematical structure.

Two-fermion spin addition

It is also instructive to recall how two spin-$\frac{1}{2}$ particles combine their spins. You may recall from QM classes that

$$\tfrac{1}{2} \otimes \tfrac{1}{2} = 1 \oplus 0,$$

which is just a fancy way of saying that out of two half-spin particles, the final net result total spin can be either 1 (triplet) or 0 (singlet). The triplet ($s = 1$) states are:

$$\left|1, 1\right\rangle = \left|\uparrow\uparrow\right\rangle, \quad \left|1, 0\right\rangle = \frac{\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle}{\sqrt{2}}, \quad \left|1, -1\right\rangle = \left|\downarrow\downarrow\right\rangle.$$

and the singlet state ($s = 0$):

$$\left|0, 0\right\rangle = \frac{\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle}{\sqrt{2}}.$$

The singlet state, for example, is the state that a pion finds itself in: two spin-1/2 quarks combined into a spin-0 state. The triplet state is essentially the state that a rho meson ρ finds itself in: two spin-1/2 quarks combined into a spin-1 state.

Helicity

Helicity is the projection of the spin onto the direction of a particle's momentum. It is a concept that is related, but not identical, to chirality. For massless particles, they are the same, but for massive particles, they are different. And they are different in a quantifiable way. Helicity is defined as

$$\hat{h} = \frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}, \qquad \hat{h}\,|\vec{p},h\rangle = h\,|\vec{p},h\rangle\,.$$

Clearly, just by inspecting the definition you should convince yourself that helicity is *not* a Lorentz-invariant quantity when the particle has a mass. This is because you can always just boost the system along the direction of motion and make the momentum be aligned or anti-aligned with the spin.

Similarly to chirality, we also call the helicity eigenstates LH and RH.

$$h = +\tfrac{1}{2}\ \text{(right-handed)}, \qquad h = -\tfrac{1}{2}\ \text{(left-handed)}.$$

When trying to be more precise, we can refer to these are *left-helical* $|\psi_-\rangle$ and *right-helical* $|\psi_+\rangle$ states, as opposed to *left-chiral* $|\psi_L\rangle$ and *right-chiral* $|\psi_R\rangle$ states.

> Helicity states are propagating states. Physical particles have well-defined helicity, but not necessarily well-defined chirality.

The relationship between helicity and chirality is fixed by the Dirac equation: the generalization of Schrödinger's equation to relativistic spin-1/2 particles. The two bases are related by a non-trivial transformation:

$$|\psi_L\rangle = \frac{1}{\sqrt{2E}}\left(\sqrt{E+p}\,|\psi_-\rangle + \sqrt{E-p}\,|\psi_+\rangle\right), \tag{3.1}$$

$$|\psi_R\rangle = \frac{1}{\sqrt{2E}}\left(\sqrt{E-p}\,|\psi_-\rangle - \sqrt{E+p}\,|\psi_+\rangle\right), \tag{3.2}$$

where $E = \sqrt{p^2+m^2}$ and $N = 2E$. The relation for antiparticles is the same, with the replacement $\psi \to \bar{\psi}$.

Another suggestive way to rewrite this is

$$|\psi_L\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1+\beta}\,|\psi_-\rangle + \sqrt{1-\beta}\,|\psi_+\rangle\right), \tag{3.3}$$

$$|\psi_R\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\beta}\,|\psi_-\rangle - \sqrt{1+\beta}\,|\psi_+\rangle\right), \tag{3.4}$$

where $\beta = \frac{p}{E}$ is the particle's velocity.

Let us consider the different regimes for these states. First, consider the stopped particle scenario, where $\beta \to 0$. In that case, the two helicities are maximally mixed together in the chiral states. This is not surprising: helicity states are fully scrambled for a particle at rest. Consequently, chirality is also fully scrambled. In the massless limit, $\beta \to 1$, the two helicities decouple completely and you will find that left-chiral states are equivalent to left-helical states, and that right-chiral states are equivalent to right-helical states.

Now, consider the ultra-relativistic regime, where $\beta \to 1 - \frac{m^2}{2E^2}$. In that case, we get

$$|\psi_L\rangle \simeq |\psi_-\rangle + \frac{m}{2p}|\psi_+\rangle\,, \tag{3.5}$$

$$|\psi_R\rangle \simeq \frac{m}{2p}|\psi_-\rangle - |\psi_+\rangle\,, \tag{3.6}$$

Alternatively, we may choose to invert these relations,

$$|\psi_-\rangle \simeq |\psi_L\rangle + \frac{m}{2p}|\psi_R\rangle\,, \tag{3.7}$$

$$|\psi_+\rangle \simeq \frac{m}{2p}|\psi_L\rangle - |\psi_R\rangle\,. \tag{3.8}$$

Discrete Symmetries. There are three important discrete symmetries in QFT that are going to help us understand the role of chirality and anti-particles in our discussion. These are:

	Parity $(\hat{P})$	**Time Reversal** $(\hat{T})$	**Charge Conjugation** $(\hat{C})$
Space	$\vec{x} \to -\vec{x}$	$\vec{x} \to \vec{x}$	$\vec{x} \to \vec{x}$
Time	$t \to t$	$t \to -t$	$t \to t$
Momentum	$\vec{p} \to -\vec{p}$	$\vec{p} \to -\vec{p}$	$\vec{p} \to \vec{p}$
Charge	$Q \to Q$	$Q \to Q$	$Q \to -Q$
States	$\lvert\psi_{L,R}\rangle \to \lvert\psi_{L,R}\rangle$	$\lvert\psi_{L,R}\rangle \to \lvert\psi_{L,R}\rangle$	$\lvert\psi_{L,R}\rangle \to \lvert\overline{\psi}_{R,L}\rangle$
Helicity	$h \to -h$	$h \to -h$	$h \to h$

Note that the action of $\hat{C}$ on states is to flip their chirality and quantum numbers, while the action of $\hat{P}$ and $\hat{T}$ is to flip their helicity. In this sense, helicity is related to the particle's relationship to spacetime, while chirality is related to the particle's internal properties.

Now, the combination $\widehat{CPT}$ is very important. It turns out that Lorentz invariance typically implies CPT invariance. Its full effect on a particle state is:

$$\widehat{CPT}\,|\psi_L, \vec{p}\rangle = |\overline{\psi}_R, \vec{p}\rangle\,, \qquad \widehat{CPT}\,|\psi_R, \vec{p}\rangle = |\overline{\psi}_L, \vec{p}\rangle\,. \tag{3.9}$$

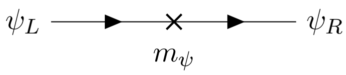
> What this means is that CPT invariance will relate left-handed particles to right-handed antiparticles. In fact, these two particles are two sides of the same coin. They come from the same field: a chiral field excites particles as well as its antiparticles with opposite chirality. For instance, the chiral field $e_L(x)$ creates $|e_L\rangle$ and $|\overline{e}_R\rangle$ states. In a consistent theory, you cannot have one without the other.

A Lorentz boost in the x direction is described by the matrix

$$\Lambda(\eta) = \begin{pmatrix} \cosh(\eta) & -\sinh(\eta) & 0 & 0 \\ -\sinh(\eta) & \cosh(\eta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.10}$$

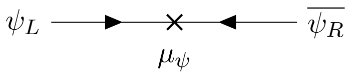
where η is the rapidity parameter related to the boost velocity by $\beta = \tanh(\eta)$. Usually we think of β and η as real parameters, but show that applying a boost with $\eta = i\pi$ and a subsequent rotation of π around the x axis, this matrix corresponds to a $\widehat{PT}$ transformation as far as the coordinates are concerned. Knowing that $\hat{C}$ acts to transform a particle to an antiparticle and that, a particle traveling backwards in time is equivalent to an antiparticle traveling forwards in time (this is the Feynman-Stueckelberg interpretation of antiparticles), conclude that the combination $\widehat{CPT}$ is analogous to this "complexified" Lorentz-invariance.

Majorana vs Dirac fermions. We saw that Dirac fermions are superpositions of two chiral states, $|\psi_L\rangle$ and $|\psi_R\rangle$. What this implies is that they require the existence of two chiral fields: one excites $|\psi_L\rangle$ and the other excites $|\psi_R\rangle$. Diagramatically, you can represent a Dirac mass as follows:



When the mass comes from the Higgs vev (as in the SM), then the mass insertion (the cross) is replaced by an interaction with the Higgs background with which weak charge can be "exchanged". In Eq. (3.5), we saw how masses scramble chiralities in the propagating states of definite helicity, requiring at least two chiral particles involved. However, in this section, I will show that a Dirac mass term is not the only way to combine two chiralities together.

Majorana fermions are particles whose mass arises from a coupling between the fermion $|\psi_L\rangle$ and its own antiparticle $|\overline{\psi}_R\rangle$. This seems reasonable: two different chiralities scramble together due to the mass term. Diagramatically, you can represent a Majorana mass as follows:



Typically, Majorana masses are denoted by the Greek letter μ to distinguish them from Dirac masses, which are usually denoted by m or m_D. The clashing arrows along the fermion line have an important consequence: electric charge is not conserved in this diagram. In fact, any quantum number associated with the particle is violated by a full two units in the presence of a Majorana mass. In the example above, electric charge was violated by a full $\Delta Q = 2$ units. This explains why, with the exception of neutrinos, all SM fermions must be Dirac particles: electric charge is conserved.

As far as we know, neutrinos are the only elementary particles that can acquire a Majorana mass.[5] For now, let us ignore the potential existence of ν_R. A SM neutrino state $|\nu_L\rangle$ combining with its own antiparticle $|\bar{\nu}_R\rangle$ would give us the two helicity states of a Majorana neutrino:

$$|\nu_-\rangle \simeq |\nu_L\rangle + \frac{m_\nu}{2p}\,|\bar{\nu}_R\rangle\,, \qquad |\nu_+\rangle \simeq -\,|\bar{\nu}_R\rangle + \frac{m_\nu}{2p}\,|\nu_L\rangle\,. \tag{3.11}$$

Note it mixes two states with opposite quantum numbers. This brings up an important question: where should we put the $^-$ sign? On ν_-? Or ν_+? This ambiguity is a reflection of the fact that the helicity states both particles and antiparticles at the same time; the distinction is formally speaking impossible to make.

> This is why you will often hear that
>
> *Majorana particles are their own antiparticles.*
>
> This sentence is unfortunately quite confusing because it omits the fact that there are still two different particles produced from the Majorana field, namely the two different helicity eigenstates. From the relation in Eq. (3.11), you can see that ultra-relativistic ν_- states will interact mostly as a *neutrino*, while ν_+ will interact mostly as an *antineutrino*. This is why we can still talk about neutrinos and antineutrinos in the context of Majorana masses: by convention, we call the *left-helical* states by neutrinos and the *right-helical* by antineutrinos. In the non-relativistic case, this distinction is far less useful, and both helicity states interact equally as neutrinos and antineutrinos.

[5] If dark matter is a fermionic particle, it may also be a Majorana particle since dark matter should be electrically neutral to have escaped detection so far.

How many different states can a Dirac fermion find itself in? How many degrees of freedom does it have? Well, firstly, the two chiral fields, $|e_L\rangle$ and $|e_R\rangle$, will produce two independent states, so at least 2. However, we saw that CPT invariance implied that each chiral field can also produce antiparticles of opposite chirality, so in total we have $n_{\text{dof}} = 4$ different states:

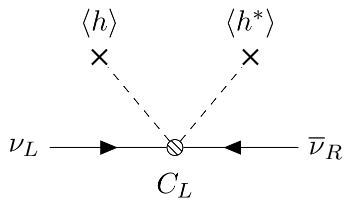
$$\psi(x)_{\text{Dirac}} = \psi_L(x) + \psi_R(x) \text{ creates } |\psi_L\rangle\,, |\overline{\psi}_R\rangle, |\psi_R\rangle, \text{ and } |\overline{\psi}_L\rangle. \tag{3.12}$$

In weak interactions, however, the Dirac field only shows up in the combination $\hat{P}_L\psi(x)_{\text{Dirac}}$ (recall Eq. (2.3)), so it can only produce the states $|\psi_L\rangle$ and $|\overline{\psi}_R\rangle$. This should be contrasted with a Majorana or Weyl fermion. There, only one chirality is present, leading to half as many degrees of freedom, $n_{\text{dof}} = 2$:

$$\psi_L(x) \text{ creates } |\psi_L\rangle \text{ and } |\overline{\psi}_R\rangle, \tag{3.13}$$

and similarly for ψ_R. But crucially, ψ_R and ψ_L are independent fields and need not both exist. A Weyl fermion has no Majorana mass, but has the same number of degrees of freedom as a Majorana fermion.

Neutrino majorana masses in the SM. At this point something should still be bothering you: how come neutrinos are allowed to have a Majorana mass if they carry weak charge? Should that also not be conserved? The answer is that it should be conserved and the Higgs background shows us how. This diagram shows how a SM neutrino Majorana mass term, μ_L, would look like:



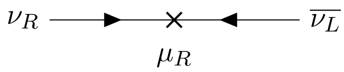
$$\longrightarrow \qquad \mu_L = \frac{C_L \times \langle h\rangle^2}{\Lambda}$$

(Majorana mass: $\nu_L \to \overline{\nu}_R$ via double Higgs vev insertion, effective vertex shown as blob.)

where C_L is a dimensionless coupling constant and Λ is some new energy scale that we will discuss shortly. Observe how we needed not one, but two(!) Higgs vev insertions to achieve this mass term. A quick inspection of the balance of Hypercharges shows why: $Y_W(\overline{\nu}_R) = -Y_W(\nu_L) = +1$ and $Y_W(h) = +1$. Note that I used the vev of the complex-conjugate of the Higgs field, h^*, which has $Y_W(h^*) = -Y_W(h)$. Its vev is the same, but I wanted to draw attention to the fact that in the way the diagram is drawn, you get an anti-Higgs insertion in the final state.

The diagram above is what we call the Weinberg mechanism for neutrino masses. It turns out that this mechanism is just an effective description of neutrino masses, and not really an explanation per se. Time permitting, we will talk about why that is the case, but for now, suffice it to say that it points to the existence of a new particle or some physics beyond the SM at an unknown scale Λ. You can convince yourself of that by noting that if diagrams with a single vev insertion $\langle h\rangle$ generate the fermion mass, then diagrams with two insertions generate a "mass squared", so to restore the dimensions to one power of mass, you need to divide by some new scale Λ. Quite literally, up to couplings and factors of 2, we have $m_\nu \sim \langle h \rangle^2/\Lambda$. What the existence of Λ is telling you is that new particles, such as RH neutrinos or new Higgs bosons, should exist at this new energy scale and complete the theory.

If the SM is extended to include RH neutrinos ν_R, then they can also have a Majorana mass term with no Higgs vev insertions at all; they are sterile fermions and do not carry any SM charge.



To summarize: we have seen two ways to extend the SM to endow neutrinos with a mass. (1) Follow the Higgs mechanism, introducing a new field $\nu_R(x)$ to make neutrinos Dirac particles. (2) Introduce a Majorana mass term for the left-handed neutrinos ν_L with a double Higgs vev insertion. The latter comes at the cost of deferring the origin of this diagram to new particles that live at some new energy scale Λ. Given that there is a whole new particle in approach (1) and that in (2) there is a new kind of mass unlike anything we ever seen in fundamental particles, you should be surprised to hear that we are very bad at telling these two scenarios apart experimentally.

The reason is due to a combination of two factors: 1) the weak force only couples to LH neutrinos (RH antineutrinos), 2) neutrinos are almost always ultra-relativistic (with one exception you will think about for your first assignment), meaning that their helicity is almost always aligned with their chirality.

Lepton number. Lepton number L is an important quantum number associated with neutrinos. As the name suggests,

$$L(\nu_L) = +1, \qquad L(\bar{\nu}_R) = -1. \tag{3.14}$$

In the same way, $L(e^-) = +1$ and $L(e^+) = -1$. All other SM particles that are not leptons do not carry lepton number, $L(\text{quarks}) = L(\text{mediators}) = L(\text{higgs}) = 0$.

> The SM happens to conserve lepton number in all of its interactions and masses. There simply is no parameter in the SM that could violate lepton number without violating other more fundamental symmetries, like electric charge conservation, so L was automatically conserved. This is an accidental symmetry: it was never imposed, but it just so happens to be true. If neutrinos are Majorana, however, it is then violated by $\Delta L = 2$ units.

Therefore, if our solution to the neutrino mass puzzle contains a Majorana mass for the left-handed neutrinos ν_L, this accidental symmetry will be no longer valid: we would expect to see processes that violate lepton number conservation.

Can you think of an example of a processes that violates lepton number conservation without violating electric charge, for example?

Note that we said that RH neutrinos ν_R can admit a Majorana mass with no major consequences. They are sterile fermions, so not weak charge is violated by it. However, they are still leptons,

$$L(\nu_R) = +1, \qquad L(\bar{\nu}_L) = -1, \tag{3.15}$$

so a Majorana mass for ν_R would still violate lepton number conservation. The consequences of it would be much harder to see experimentally because ν_R do not interact (directly) with anything in the SM, but as we will see, it can still have important consequences as ν_R can transfer some of this lepton number violation to the LH neutrino states, ν_L, through the Higgs interaction.

In particle physics we operate under the motto:

All that is not forbidden is mandatory.

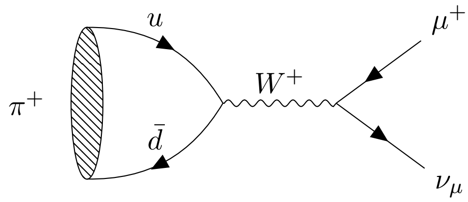
So if we do not enforce that lepton number conservation is a symmetry law, then by simply allowing ν_R to exist, we can expect it to have a Majorana mass. This mass can be anything, as, a priori, it has nothing to do with the Higgs boson or its vev.

Therefore, trying to include neutrino masses in the SM by adding ν_R neutrinos to the theory is not so innocuous: once introduced, you must make a choice. Does it have a Majorana mass or not? If not, then that's significant: we just learned about another symmetry law. If it does, then the two chiral neutrino states ν_L and ν_R want to pair up into a Dirac fermion but the Majorana mass of ν_R prevents this from happening.

Note that ν_L may also carry a Majorana mass *à la* the Weinberg interaction discussed above. But in that case, it is less clear whether such a mass must exist or not. This is because it relied on the existence of this new physics scale Λ, which we know absolutely nothing about. If there are no new particles at this scale, such a Weinberg mechanism is never triggered and we were never justified in writing it down. If this was the only way to give neutrino masses, then we could be more confident that such a mass scale exists, but we already know that Dirac neutrino masses can solve the neutrino mass puzzle without invoking any Majorana mass terms.

We will return to this interplay between Dirac and Majorana masses. But first, we must convince ourselves that the data is really telling us that neutrinos have mass, so we will wait until we have studied neutrino oscillations in detail.

Neutrino to anti-neutrino conversion. How hard is it exactly to determine if a neutrino state is Dirac or Majorana? One way to distinguish these two scenarios is to ask if neutrinos can violate lepton number in their interactions. If they can, then that is a pretty strong indication that they are Majorana particles. Start with a neutrino source, say, pion decay $\pi^+ \to \mu^+ + \nu_\mu$:



(Pion decay diagram: The pion, represented by the blob to emphasize its compositeness, can decay once its constituent quarks come together to annihilate into a virtual W^+ boson, which in turn decays into a muon and a muon neutrino.)

Because this is a weak interaction process, this decay proceeds through the interactions of a right-handed anti-muon $|\mu_R^+\rangle$ and a left-handed muon neutrino $|\nu_L\rangle$.[6] Now, we know that the propagating degrees of freedom are helicity states, and not chirality states. So, when the pion decays, we must project the interaction states onto the propagating degrees of freedom to find the actual helicity states produced. Schematically, the probabilities to decay into the two states are

$$P(\pi^+ \to \mu^+\nu_-) \propto |\langle \nu_- | \nu_L \rangle|^2 \simeq 1 - \frac{m^2}{4E^2}, \tag{3.16}$$

$$P(\pi^+ \to \mu^+\nu_+) \propto |\langle \nu_+ | \nu_L \rangle|^2 = \frac{m^2}{4E^2}. \tag{3.17}$$

What this shows is that the pion actually decays mostly to $|\nu_-\rangle$ states and a tiny amount of $|\nu_+\rangle$ states because of Eq. (3.5). To be clear: the pion decays to two separate populations. The pion does *not* decay to the superposition $|\nu_L\rangle$, but rather, each time the pion decays, it produces either $|\nu_-\rangle$ or $|\nu_+\rangle$ with the aforementioned proportions. This is because the amplitude for producing these two states do not interfere with each other.

Now, if neutrinos are Majorana, this small amount of $|\nu_+\rangle$ is quite significant. It means, by virtue of Eq. (3.11), that a small fraction of neutrinos produced in pion decay actually prefers to interact as an antineutrino, $|\bar{\nu}_R\rangle$. To see this, consider the left-helical states $|\nu_-\rangle$. These are almost identical to left-chiral neutrinos: $|\nu_L\rangle$, which through the weak force, scatter on nucleons as follows:

$$\nu_L + n \to \mu^- + p^+, \qquad P(\nu_- + n \to \mu^- + p^+) = |\langle \nu_- | \nu_L \rangle|^2 \simeq 1 - \frac{m^2}{4E^2}. \tag{3.18}$$

Now, the right-helical state $|\nu_+\rangle$ for Majorana neutrinos is almost identical to $|\bar{\nu}_R\rangle$, which also interacts with the weak force, but it does so as an antiparticle. The scattering on nucleons is then strikingly different:

$$\bar{\nu}_R + p^+ \to \mu^+ + n, \quad P(\bar{\nu}_+ + p^+ \to \mu^+ + n) = |\langle \nu_+ | \bar{\nu}_R \rangle|^2 \simeq 1 - \frac{m^2}{4E^2}. \tag{3.19}$$

This explains why we typically refer to ν_+ as an antineutrino.

Now, using only the helicity eigenstates, if you keep track of lepton number in these reactions, you will find that there are four possible chains of events, shown in Fig. 1. The

[6] For now, we will drop the flavor subscript ν_μ and simply write ν for simplicty.

first conserves L overall,

$$\pi^+ \to \mu^+ + \nu_- \quad \text{followed by} \quad \nu_- + n \to \mu^- + p^+ \quad (\Delta L = 0). \tag{3.20}$$

Overall, this process starts with a pion, produces a μ^+ and a ν_-. The latter has no well-defined lepton number (it is a combination of a neutrino and an antineutrino), so to determine the overall lepton number in the full process, we will ask what charged lepton gets produced once it scatters, which, in this case, is a μ^- So, starting with the pion, which had no lepton number, we ended up with a μ^+ and a μ^-, which have opposite lepton numbers and therefore, no net gain or loss of L took place. Note that the neutron is there just to serve as a target for the neutrino and also does not change the overall lepton number.

Now, let us consider a chain of events that violates L (by 2 units, as we expect from an effect that comes from the Majorana mass term!),

$$\pi^+ \to \mu^+ + \nu_+ \quad \text{followed by} \quad \nu_+ + p^+ \to \mu^+ + n \quad (\Delta L = 2,\ \text{suppressed by } m^2/4E^2). \tag{3.21}$$

Overall, this second chain sees the production of two μ^+ particles: one from pion decay, and the other from the neutrino scattering on a nucleon. The suppression factor $m^2/4E^2$ comes from the small overlap of ν_L, the interaction basis neutrino, with ν_+, namely $\langle \nu_L | \nu_+ \rangle \simeq \mathcal{O}(m^2/E^2)$. But once we pay this penalty for the production of neutrinos of the "wrong helicity" out of pion decay is suppressed, the scattering can proceed as shown with large probability because the overlap $\langle \bar{\nu}_R | \nu_+ \rangle \simeq 1$.

The two remaining possibilities to obtain the same processes above are as follows: For the L violating one, you could have also asked that ν_- interact as a right-chiral *antineutrino* $|\bar{\nu}_R\rangle$. This is again suppressed by an overall factor of $m^2/4E^2$, which comes from the scattering process, rather than from the decay. And the last possibility, and least important one, is to require that the pion decay to ν_+ and but force it to interact as a left-chiral *neutrino* $|\nu_L\rangle$. In this case, the overall suppression is of order $m^4/16E^4$, so completely negligible.

So, how likely are we to observe such lepton number violating processes? The suppression factor in any realistic experiment is of order

$$\frac{m^2}{4E^2} \simeq 2.5 \times 10^{-21} \left(\frac{m_\nu}{0.1\ \text{eV}} \right)^2 \left(\frac{1\ \text{GeV}}{E_\nu} \right)^2 . \tag{3.22}$$

This is telling you that to see a single μ^+ in a beam of neutrinos from π^+ decay, you would need to detect about 10^{21} neutrinos in total, which is completely unrealistic. Typically, neutrino experiments detect about 10^6 neutrinos in total.

> What if we consider neutrinos from neutron beta decay instead? How many ν_e interactions would you need to detect to see a single lepton number violating event in that case?

The story above only holds for Majorana neutrinos. If they are Dirac, the $m^2/4E^2$ suppressed piece of these processes is far less significant. For ultra-relativistic Dirac neutrinos, the "wrong-helicity" $|\nu_+\rangle$ component produced in pion decays is approximately $|\nu_R\rangle$. Therefore, it does not interact through the weak force (again, up to $\mathcal{O}(m^2/4E^2)$ corrections that contain $|\nu_L\rangle$ — which is still a *neutrino*). This means that the chains of events above that lead to lepton number violation are impossible for Dirac neutrinos: this had to be the case since Dirac neutrino masses conserve lepton number. The only effect of the suppressed amplitudes are to reduce the overall interaction rate of neutrinos by a tiny amount, which is even more challenging to measure experimentally.

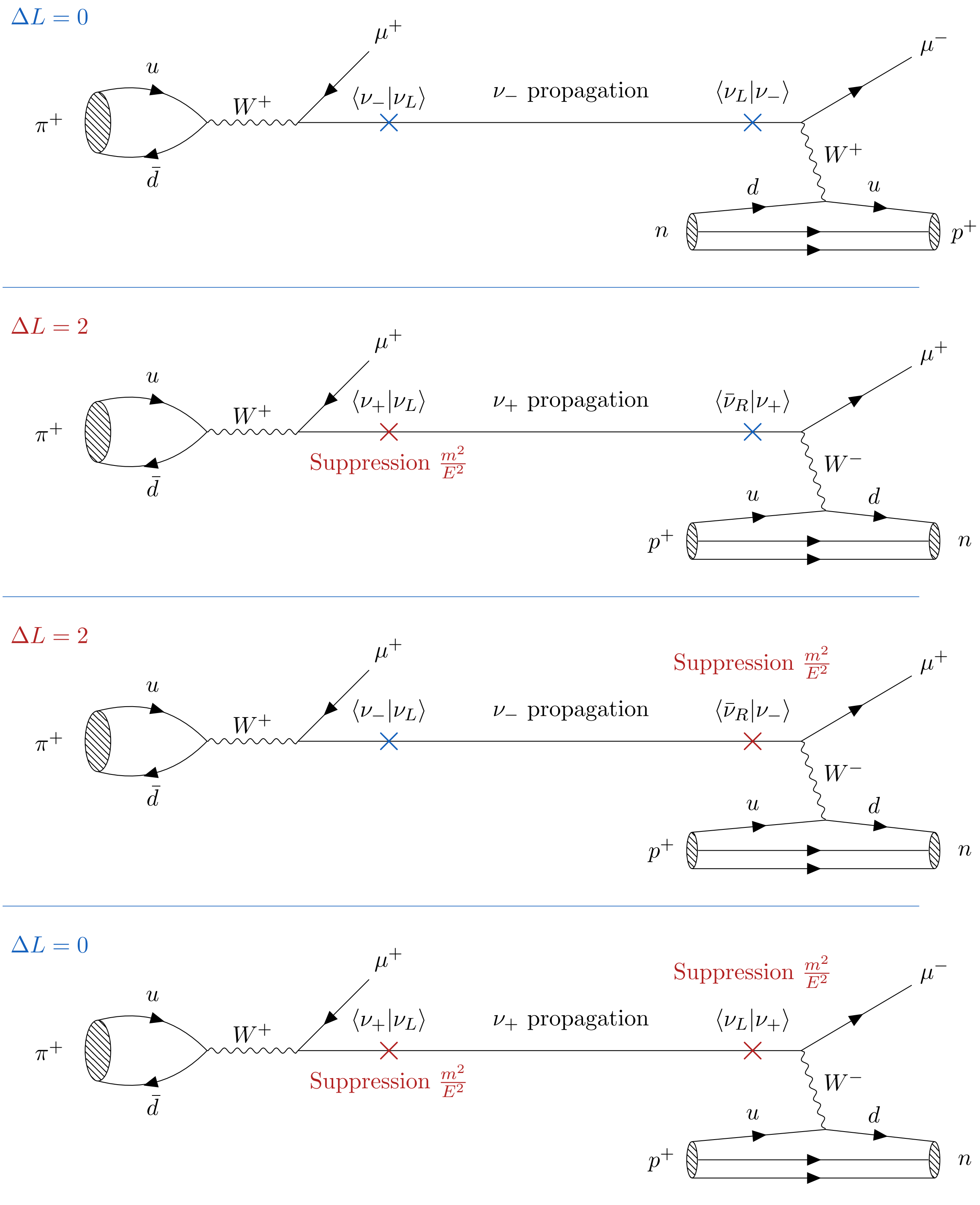


Figure 1: Pion decay to neutrinos followed by neutrino scattering on a nucleon target for different possibilities under a Majorana neutrino scenario. Spectator quarks are shown to emphasize that the nucleon is a bound state.